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# The Macroeconomic Dynamics of Demographic Shocks

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## Abstract

The paper employs an extended Yaari-Blanchard model of overlapping generations to study how the macroeconomy is affected over time by various demographic changes. It is shown that a proportional decline in fertility and death rates has qualitatively similar effects to capital income subsidies; both per capita savings and per capita consumption increase in the new steady state. A drop in the birth rate, while keeping the death rate constant, reduces per capita savings, but increases per capita consumption, particularly if intertemporal labor supply is very elastic. If the generational turnover effect is sufficiently strong, however, a decline in the birth rate may, contrary to standard results, give rise to an increase in per capita savings. Finally, a fertility rate reduction which leaves unaffected the rate of generational turnover is shown to have effects qualitatively similar to those of a fall in public consumption. Both per capita savings and per capita output decline, but per capita consumption rises. The non-linear model is simulated to study the quantitative effects of non-infinitesimal demographic shocks.

**JEL codes:** E12, E63, L16.

**Keywords:** fertility rate, intertemporal labor supply, overlapping generations, Blanchard model, demographic shocks, transition effects.

## 1 Introduction

Population aging and its macroeconomic effects have emerged over the last decade as a key issue on the policy agendas of most industrialized countries. During the post-war period, the population share of elderly people has increased dramatically. Following the post-war “baby

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boom”—during which population growth rates temporarily accelerated—fertility rates have declined substantially, commonly known as the “baby bust.” At the same time, mortality rates have decreased in most industrialized nations, owing to healthier lifestyles and medical advances.<sup>1</sup> Both trends give rise to population aging.

The effects of the post-war demographic transition on old-age dependency ratios (that is, the ratio of the population aged 65 years and older to the population aged 15-64 years) for selected OECD countries are presented in Table 1. The evolution of the old-age dependency ratio shows pronounced population aging for all countries, where it is apparent that populations in European countries and Japan are older than elsewhere. Japan stands out as having an old-age dependency ratio of only 10 percent in 1970, and a projected ratio of more than 70 percent in 2050, a large increase unparalleled across OECD countries. The youth dependency ratio<sup>2</sup> is projected to fall by about one third between 1970 and 2050, reflecting a steady decline in fertility rates, although there are significant differences among OECD countries. Canada and New Zealand experience a decline in the youth dependency ratio of about 26 percentage points, against a fall of only 9 percentage points in Japan.

Demographic changes have profound economic effects, which may span many generations. Particularly, the impending retirement of the baby boom generation is raising a great deal of concern. If a large fraction of the population retires (or passes away), society is expected to save less, leading to a lower rate of capital accumulation and lower living standards.<sup>3</sup> The aim of the paper is to analyze the macroeconomic effects of various demographic changes in a model of a closed economy. Various questions arise. How do changes in the population growth rate affect aggregate savings, consumption, employment and output in the new steady state? Does a drop in fertility, reducing potential labor supply, drive up wages? How are the relevant macroeconomic variables affected along the transition path?

The informal literature on population aging, for example, Group of Ten (1998) and Mc-Morrow and Röger (2003), is voluminous. Many formal contributions employ calibrated life-cycle models—in the tradition of Samuelson (1958), Diamond (1965)<sup>4</sup> and Auerbach and Kotlikoff (1987)—to study numerically the effects of population aging.<sup>5</sup> All of these studies assume exogenous population dynamics. Some authors employing the life-cycle approach—for

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<sup>1</sup>Birth rates (per 100 of the population) in the United States came down from 2.43 in 1950 to 1.45 in 2000. The drop in death rates (per 100 of the population) was less spectacular, declining from 0.95 in 1950 to 0.83 in 2000 (United Nations, 2003).

<sup>2</sup>The youth dependency ratio is defined as the ratio of the population aged 0-14 years to the population aged 15-64 years.

<sup>3</sup>Bloom, Canning and Graham (2003) provide econometric evidence on the positive relationship between life expectancy and the savings rate.

<sup>4</sup>Diamond (1965) assumes that individuals live for two discrete time periods, in which they work and save in the first period and consume out of savings in the second period.

<sup>5</sup>Auerbach and others (1989), Cutler and others (1990), Auerbach, Cai and Kotlikoff (1991), Ríos-Rull (2001), and Brooks (2002) employ a Diamond-Samuelson overlapping generations model, which is generalized to many periods.

**Table 1. Dependency Ratios for Selected OECD Countries,  
1970-2050 (In Percent)**

Country	Youth Dependency Ratios <sup>(1)</sup>				Old-Age Dependency Ratios <sup>(2)</sup>			
	1970	2000	2025 <sup>(3)</sup>	2050 <sup>(3)</sup>	1970	2000	2025 <sup>(3)</sup>	2050 <sup>(3)</sup>
Belgium	37	27	25	26	21	26	37	47
Canada	49	28	23	26	13	18	33	44
Denmark	36	27	25	26	19	22	35	42
France	40	29	27	28	21	24	36	46
Germany	37	23	22	27	22	24	38	49
Italy	38	21	18	25	17	27	40	65
Japan	35	21	20	26	10	25	50	72
Netherlands	44	27	25	26	16	20	33	42
New Zealand	53	35	29	27	14	18	29	38
Spain	45	21	18	26	16	24	35	68
Switzerland	37	25	22	24	17	24	45	55
United Kingdom	39	29	25	26	21	24	31	38
United States	46	33	32	29	16	19	28	32

Source: United Nations (2003), World Population Prospects Database. (1) The ratio of the population aged 0-14 to the population aged 15-64. (2) The ratio of the population aged 65 years or over to the population aged 15-64. (3) Medium variant projections.

example, Elmendorf and Sheiner (2000)—have examined the steady-state effects of demographic changes analytically, but have not studied the entire transition path.<sup>6</sup> Our analysis follows a different take. It draws on the overlapping generations framework of Yaari (1965) and Blanchard (1985), which assumes that all agents face a constant probability of death in a world of zero population growth, as well as Buiter (1988), who introduces (exogenous) population growth in the Yaari-Blanchard model.<sup>7</sup> In particular, we extend Buiter’s model to include endogenous (intertemporal) labor supply, which allows us to study the labor market effects of various demographic shocks. By explicitly modeling the labor market we can disentangle labor supply responses—and thus the participation decision—from labor demand conditions, thus allowing for a meaningful analysis of intertemporal wage profiles. Moreover, we can allow for voluntary retirement of households by incorporating a wealth effect in labor supply that makes old agents—having accumulated much wealth—work less hours. Our framework is partly related to the model of Heijdra and Ligthart (2000, 2002), which has introduced endogenous labor supply in the Yaari-Blanchard framework to study taxation issues.<sup>8</sup>

A simple graphical apparatus is developed to provide an intuitive account of the long-run and dynamic effects of various demographic changes. The model is versatile because it encompasses results of various seminal works—that of Blanchard (1985), Buiter (1988), and Weil (1989)—by varying the assumptions made on demography and the intertemporal substitution elasticity in labor supply. Moreover, it can be employed to get insight into and extend the results from the population aging literature. Our approach differs from previous theoretical analyses on population dynamics by being able to trace out impulse-responses at business cycle frequencies.<sup>9</sup> In the Samuelson-Diamond framework, however, a typical period lasts 35 years, implying that transitional dynamics can only be studied at low frequencies. Knowledge of the entire transition path is of importance to policy analysis, however, because the short-run effects of demographics shock differ markedly from their long-run effects.

Three demographic scenarios are analyzed analytically. The first shock concerns an unexpected and permanent decrease in the (exogenous) fertility rate (that is, a pure “baby bust”). It is shown that the optimal savings response to declining fertility entails either a decrease or an increase in per capita savings depending on the assumptions made on the elasticity of labor supply and the generational turnover effect, thereby generalizing the results of Elmendorf and

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<sup>6</sup>Momota and Futagami (2000), however, study demographic transition in a small open economy using endogenous fertility theory (see Becker and Barro (1988)).

<sup>7</sup>Weil (1988) also allows for population growth, but he assumes infinitely-lived overlapping generations and thus differs from the uncertain lifetimes approach of Yaari-Blanchard.

<sup>8</sup>Bovenberg (1993) has employed the Yaari-Blanchard-Buiter framework to study the effects of a permanent rise in the capital tax in an open economy. Bovenberg and Heijdra (1998), however, consider a closed economy, but they do not allow for net population growth.

<sup>9</sup>The Laplace transform technique of Judd (1982) is used to solve for the entire transition path of the demographic change.

Sheiner (2000). Second, we study a proportionate fall in the fertility and death rates so as to yield a stationary population growth rate. Under this scenario, the qualitative results are identical to those of a subsidy on capital; both the per capita capital stock and per capita consumption rise in the new steady state. The final scenario—studying a drop in the fertility rate and a compensating increase in the death rate so as to maintain the rate of generational turnover constant—gives rise to a rise in the long-run per capita consumption, although the per capita capital stock falls.

Then, the paper proceeds with a quantitative analysis of non-infinitesimal demographic shocks. Numerical results show that the macroeconomic effects of shocks are of first-order nature; most elasticities are well above unity. Furthermore, we study what kind of stabilization policy should be devised to counteract the effects of various demographic changes. Attention is restricted to public consumption and capital taxes as available instruments of fiscal policy.<sup>10</sup> It is demonstrated that depending on the nature of the demographic change, a different fiscal policy is needed to neutralize the effect of the shock. A drop in fertility under constant population growth can be offset by positive capital taxes, whereas a mix of higher capital taxes and more public consumption is needed to address a pure baby bust.

The remainder of the paper is organized as follows. Section 2 sets out the Yaari-Blanchard overlapping generations model extended for endogenous labor supply and exogenous population growth. Section 3 solves the model graphically and analyzes the dynamics around the long-run equilibrium. Section 4 employs the graphical framework of Section 3 to qualitatively study various demographic shocks. Section 5 presents numerical simulation results with a view to obtain insights in the quantitative effects of the shocks. Section 6 concludes.

## 2 A Model of Overlapping Generations

### 2.1 Individual Households and Demographics

As in Blanchard (1985), individual households face an age-invariant probability of death ( $\beta \geq 0$ ). Each household has a time endowment of unity, which is allocated optimally over labor supply and leisure. The utility functional at time  $t$  of the representative agent born at time  $v$  is denoted by  $\Lambda(v, t)$ :

$$\Lambda(v, t) \equiv \int_t^\infty [\varepsilon_C \log \bar{c}(v, \tau) + (1 - \varepsilon_C) \log [1 - \bar{l}(v, \tau)]] e^{(\alpha + \beta)(t - \tau)} d\tau, \quad (1)$$

where  $\bar{c}(v, t)$  and  $\bar{l}(v, t)$  are, respectively, private consumption and labor supply in period  $t$  by an agent born in period  $v$ ,  $\alpha$  is the pure rate of time preference ( $\alpha > 0$ ) that applies across generations and  $\varepsilon_C$  is the share of consumption in utility.<sup>11</sup> The logarithmic felicity function

<sup>10</sup>For simplicity, labor taxes and government debt have been abstracted from.

<sup>11</sup>We use the notation introduced by Buiter (1988) by letting lowercase barred variables denote values at the individual household level.

implies that the intertemporal substitution elasticity for goods consumption is unity and for labor supply is  $[1 - \bar{l}(v, t)] / \bar{l}(v, t)$ . The representative agent's dynamic budget identity can be expressed as:

$$\dot{\bar{a}}(v, t) = [r(t) + \beta] \bar{a}(v, t) + w(t) \bar{l}(t) - \bar{z}(t) - \bar{c}(v, t), \quad (2)$$

where  $\dot{\bar{a}}(v, t) \equiv d\bar{a}(v, t)/d\tau$ ,  $\bar{a}(v, t)$  are real financial assets,  $r(t)$  is the real rate of interest,  $w(t)$  is the real wage rate (assumed age-independent for convenience), and  $\bar{z}(t)$  are real net lump-sum taxes. The return on financial assets exceeds the rate of interest because, with life-time uncertainty and in the absence of bequest motives, agents conclude actuarially fair contracts with life insurance companies.<sup>12</sup>

The individual household chooses time profiles for  $\bar{c}(v, t)$  and  $\bar{l}(v, t)$  in order to maximize  $\Lambda(v, t)$  subject to the budget identity (2) and a No-Ponzi-Game solvency condition,  $\lim_{\tau \rightarrow \infty} \bar{a}(v, \tau) \exp[-\int_t^\tau [r(s) + \beta] ds] = 0$ . The optimal solutions for private consumption and labor supply on the interval  $t \in [0, \infty)$  are fully characterized by:<sup>13</sup>

$$\bar{c}(v, t) = \varepsilon_C (\alpha + \beta) [\bar{a}(v, t) + \bar{h}(t)], \quad (3)$$

$$1 - \bar{l}(v, t) = \frac{(1 - \varepsilon_C) \bar{c}(v, t)}{\varepsilon_C w(t)}, \quad (4)$$

$$\frac{\dot{\bar{c}}(v, t)}{\bar{c}(v, t)} = r(t) - \alpha, \quad (5)$$

where  $\bar{h}(t)$  is expected lifetime human wealth:

$$\bar{h}(t) \equiv \int_t^\infty [w(\tau) - \bar{z}(\tau)] \exp \left[ - \int_t^\tau [r(s) + \beta] ds \right] d\tau, \quad (6)$$

which is age independent. According to (3) goods consumption in the planning period  $t$  is proportional to total wealth, comprising the sum of financial and human wealth. Equation (4) shows that in each period, the marginal rate of substitution between leisure and private consumption is equated to the wage rate. Note that labor supply is a negative function of individual consumption. This wealth effect causes wealthier agents to consume more leisure and thus allows for the proportion of agents opting for “voluntary retirement” to increase with age.<sup>14</sup> As the Euler equation (5) shows, the time profile of individual consumption is governed by the difference between the real interest rate and the rate of pure time preference.

<sup>12</sup>In particular, agents receive an annuity payment from the insurance company proportional to their financial wealth ( $\beta \bar{a}(v, t)$ ) in exchange for transferring their entire estate to the insurance company upon death. Since the contracts are actuarially fair, the annuity rate equals the death rate  $\beta$ .

<sup>13</sup>Details of the solution methods and all mathematical derivations can be found in a technical appendix (Heijdra and Ligthart, 2004), which is available upon request.

<sup>14</sup>Because there is no upper limit on an agent's age, theoretically the possibility cannot be excluded that there exists some arbitrarily wealthy agent wanting to consume more leisure than its time endowment allows for (that is,  $\bar{l}(v, t) < 0$  for  $v \rightarrow -\infty$ ).

Finally, equation (6) implies that human wealth is the after-tax value of the time endowment discounted at the risk-of-death adjusted rate of interest  $r + \beta$ .

To allow for net population growth or decline, we draw on Buiter (1988) and distinguish between the probability of death  $\beta$  ( $\geq 0$ ) and the birth rate  $\eta$  ( $\geq 0$ ). An attractive feature of modelling demographics this way is that it nests two seminal overlapping generations models as special cases. By setting  $\eta = \beta$  Blanchard's (1985) model<sup>15</sup> is obtained and by setting  $\beta = 0$  Weil's (1989) model is derived.<sup>16</sup> Without international migration, the (net) population growth rate,  $n_N$ , equals the difference between the birth and death rate:<sup>17</sup>

$$n_N \equiv \frac{\dot{N}(t)}{N(t)} = \eta - \beta \begin{cases} \leq 0, & \text{if } \eta \leq \beta, \\ > 0, & \text{if } \eta > \beta, \end{cases} \quad (7)$$

where the population size at time  $t$  is denoted by  $N(t)$ . The size of a newborn generation is proportional to the current population  $N(v, v) = \eta N(v)$ , where  $N(v, t)$  is the size at time  $t$  of the cohort born at some time  $v$  ( $t \geq v$ ). Since the death rate is constant and cohorts are assumed to be large, the size of each existing generation falls exponentially according to:

$$N(v, t) = e^{-\beta(t-v)} N(v, v), \quad t \geq v. \quad (8)$$

## 2.2 Aggregate Household Sector

Given the simple demographic structure, aggregate variables can be calculated as the weighted integral of the values for the different generations. Aggregate financial wealth is, for example, defined as  $A(t) \equiv \int_{-\infty}^t N(v, t) \bar{a}(v, t) dv$ , where  $N(v, t) = \eta e^{\eta v} e^{-\beta t}$  (and aggregate values for  $C(t)$  and  $L(t)$  are derived in a similar fashion). The main equations describing optimal behavior of the aggregate household sector can be written as:

$$C(t) = \varepsilon_C (\alpha + \beta) [A(t) + N(t) \bar{h}(t)], \quad (9)$$

$$N(t) - L(t) = \frac{(1 - \varepsilon_C)C(t)}{\varepsilon_C w(t)}, \quad (10)$$

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \alpha - \left[ \frac{\beta C(t) - \eta N(t) \bar{c}(t, t)}{C(t)} \right]. \quad (11)$$

Equations (9) and (10) are aggregate versions of (3) and (4), respectively. Equation (11) is the Keynes-Ramsey rule modified for the existence of overlapping generations of finitely-lived agents. It says that aggregate consumption growth differs from individual consumption growth

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<sup>15</sup>Furthermore, the intertemporal labor supply elasticity should equal unity because labor supply is exogenous in Blanchard's model. See the Appendix for further details.

<sup>16</sup>Bovenberg (1993) interprets the special case of  $\eta = 0$  and  $\beta < 0$  as a Ramsey model with *intra*-dynasty population growth, implying that the Ricardian equivalence proposition still holds. If  $\eta > 0$  and  $\beta = 0$ , there is *extra*-dynasty growth. Due to the birth of disconnected generations (or dynasties) Ricardian equivalence would not hold.

<sup>17</sup>By solving (7) subject to the initial condition  $N(0) = 1$ , the path for the aggregate population is obtained:  $N(t) = e^{n_N t}$ .



(equation (5)), owing to the distributional effects caused by the turnover of generations. This so-called *generational turnover* effect (cf. Heijdra and Ligthart (2000))—represented by the second term between brackets of (11)—is comprised of two opposing forces. On the one hand, aggregate growth exceeds individual growth, because of the birth of *new* generations, who start consuming out of human wealth immediately (represented by  $\eta N(t) \bar{c}(t, t)$ ). On the other hand, aggregate consumption growth falls short of individual growth, reflecting that at each instant of time a cross section of the population *dies* and consequently ceases to consume (represented by  $\beta C(t)$ ). For future reference, equation (11) can be rewritten in terms of aggregate variables:<sup>18</sup>

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \alpha + n_N - \eta \varepsilon_C (\alpha + \beta) \left[ \frac{A(t)}{C(t)} \right]. \quad (12)$$

### 2.3 Firms

Firms in the final goods sector<sup>19</sup> rent capital,  $K(t)$ , and labor,  $L(t)$ , from households to produce a homogeneous good,  $Y(t)$ , which is either consumed by households or the government or invested by households to augment the physical capital stock. The final goods sector is characterized by perfect competition. Technology is described by a Cobb-Douglas production function:

$$Y(t) = \Psi_Y K(t)^{\varepsilon_K} L(t)^{1-\varepsilon_K}, \quad 0 < \varepsilon_K < 1, \Psi_Y > 0, \quad (13)$$

where  $\Psi_Y$  is a general technology index, which is assumed to be constant. Real profits of the representative firm are defined in the usual way:

$$\Pi(t) \equiv (1 - \tau(t)) [Y(t) - w(t) L(t)] - [r(t) + \delta] K(t), \quad (14)$$

where  $r(t) + \delta$  is the effective rental rate of capital,  $\delta$  is the rate of capital depreciation, and  $\tau(t)$  is a capital income tax (or capital subsidy if  $\tau(t) < 0$ ). The representative producer chooses  $K(t)$  and  $L(t)$  in order to maximize  $\Pi(t)$ , taking factor prices as given. The first-order conditions for this static optimization problem are:

$$(1 - \tau(t)) \frac{\partial Y(t)}{\partial K(t)} = r(t) + \delta, \quad \frac{\partial Y(t)}{\partial L(t)} = w(t). \quad (15)$$

Since technology features constant returns to scale and markets are perfectly competitive, excess profits are zero (that is,  $\Pi(t) = 0$ ). Furthermore, since there are no adjustment costs associated with investment, the value of household share holdings equals the capital stock, that is,  $V(t) = K(t)$ .

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<sup>18</sup>Using (3), (7), and (9) and noting that, in the absence of bequests, newborns possess no financial wealth (so that  $\bar{a}(t, t) = 0$ ).

<sup>19</sup>There are many identical firms and, for convenience, their number is normalized to unity.

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**Table 2. The Model<sup>(§)</sup>**

(a) *Dynamic equations:*

$$\dot{k}(t) = y(t) - c(t) - g(t) - (\delta + n_N) k(t), \quad (\text{T1.1})$$

$$\dot{c}(t) = [r(t) - \alpha] c(t) - \eta \varepsilon_C (\alpha + \beta) k(t), \quad (\text{T1.2})$$

(b) *Static equations:*

$$y(t) = \Psi_Y k(t)^{\varepsilon_K} l(t)^{1-\varepsilon_K}, \quad (\text{T1.3})$$

$$w(t) = (1 - \varepsilon_K) \left( \frac{y(t)}{l(t)} \right), \quad (\text{T1.4})$$

$$\frac{r(t) + \delta}{1 - \tau(t)} = \varepsilon_K \left( \frac{y(t)}{k(t)} \right), \quad (\text{T1.5})$$

$$w(t) [1 - l(t)] = \left( \frac{1 - \varepsilon_C}{\varepsilon_C} \right) c(t), \quad (\text{T1.6})$$

$$\bar{z}(t) = g(t) - \tau(t) [y(t) - w(t)l(t)]. \quad (\text{T1.7})$$

(§) *Variables:*  $y(t) \equiv Y(t)/N(t)$ : per capita output;  $c(t) \equiv C(t)/N(t)$ : per capita consumption;  $g(t) \equiv G(t)/N(t)$ : per capita government consumption;  $k(t) \equiv K(t)/N(t)$ : per capita capital stock;  $l(t) \equiv L(t)/N(t)$ : per capita labor supply (that is, the macroeconomic participation rate);  $\bar{z}(t)$ : per capita lump-sum tax;  $w(t)$ : real wage rate;  $r(t)$ : real interest rate; and  $\tau(t)$ : capital income tax. *Parameters:*  $\delta$ : rate of capital depreciation;  $\alpha$ : pure rate of time preference;  $n_N \equiv \eta - \beta$ : net population growth rate;  $\eta$ : birth rate;  $\beta$ : death rate;  $\varepsilon_K$ : capital share in output; and  $\varepsilon_C$ : share of consumption in utility.

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## 2.4 Government and Market Equilibrium

The government consumes a fixed share of the final good. Abstracting from public debt and labor taxes, the periodic budget restriction of the government can be written as:

$$G(t) = N(t)\bar{z}(t) + \tau(t)[Y(t) - w(t)L(t)], \quad (16)$$

where  $G(t)$  denotes public consumption and  $N(t)\bar{z}(t)$  are total net lump-sum taxes.

Because of the assumption of perfect foresight, agents' behavior depends on current and future prices. Flexible factor prices cause factor markets to clear instantaneously. Financial market equilibrium implies that households' claims on capital equal the physical capital stock (that is,  $A(t) = K(t)$ ). Equilibrium on the goods market implies that:

$$Y(t) = C(t) + G(t) + I(t), \quad (17)$$

where  $I(t)$  denotes gross investment:

$$\dot{K}(t) = I(t) - \delta K(t), \quad (18)$$

where  $\dot{K}(t) \equiv dK(t)/dt$  is net capital accumulation.

## 2.5 Model Summary

In the presence of population growth, the model will give rise to ongoing economic growth also in the steady state.<sup>20</sup> In order to study the steady-state dynamics, we rewrite the model in a stationary format by expressing all growing variables relative to the *population size*,  $N(t)$ . The key equations of the model are presented in Table 2. Consider first the dynamic equations. Equation (T1.1) describes the evolution of the per capita stock of capital, which is obtained by combining (17) and (18) and dividing by the population size. The second equation, given by (T1.2), shows the optimum time path of per capita consumption. With a positive birth rate ( $\eta > 0$ ), the steady-state rate of interest must exceed the pure rate of time preference, that is,  $r > \alpha$ .<sup>21</sup> The rising individual consumption profile (see (5)) ensures that in steady state financial wealth is transferred—via the life-insurance companies—from old to young generations (see Blanchard (1985)).

Equations (T1.3)-(T1.7) are essentially static equations. Equation (T1.3) is the intensive-form production function, obtained from (13). The factor demand equations (T1.4)-(T1.5) are derived by rewriting the expressions in (15) in intensive form. Equation (T1.4) represents a downward sloping labor demand curve in the  $(w, l)$  space. The per capita labor supply

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<sup>20</sup>Growth is exogenous in the steady state, but endogenous during transition. See Sections 3 and 4.

<sup>21</sup>This follows from noting that the steady-state aggregate stock of financial assets (or capital stock per capita) is positive ( $k(t) > 0$ ). If the birth rate is zero (that is,  $\eta = 0$ ), then (T1.2) implies the familiar Ramsey result of  $r = \alpha$  in the steady state.

expression (T1.6) results upon rewriting (10), and is referred to as the macroeconomic participation rate. Population growth affects the participation rate both through its effect on the population size and through its effect on aggregate labor supply. Note that the participation rate is a negative function of per capita consumption. The short-run per capita capital supply curve is a vertical schedule—representing a given capital stock—whereas the short-run demand for capital (T1.5) is a standard downward sloping demand curve, owing to diminishing returns to capital accumulation. Finally, the government budget restriction (T1.7) is a reworked version of (16).

### 3 Graphical Solution

As was shown in the previous section, the dynamic part of the model can be analytically reduced to two variables: the per capita capital stock (a predetermined variable) and the per capita consumption (a forward-looking or jump variable). The model can be graphically summarized by a phase diagram as shown in Figure 1.<sup>22</sup> The figure is drawn while holding constant the  $g/y$  share. The  $\dot{k} = 0$  line represents all combinations of  $c$  and  $k$  for which the per capita stock of capital is constant over time. It passes through the origin and is upward sloping provided  $k$  falls short of its golden-rule level. For points above (below) the  $\dot{k} = 0$  line, employment<sup>23</sup> is too low (high) and consumption is too high (low) so that the capital stock falls (rises), which is indicated by the horizontal arrows in Figure 1.

The  $\dot{c} = 0$  line denotes all  $(c, k)$ -combinations for which per capita consumption is constant over time. The dashed line connecting points  $P_3$  and  $P_4$  in Figure 1 is the  $\dot{c} = 0$  line for the special case of a zero birth and mortality rate (that is,  $\eta = \beta = 0$ ), known in the literature as the infinite-horizon Ramsey (1928) model.<sup>24</sup> In this case, the steady-state rate of interest is pinned down by the pure rate of time preference (that is,  $r = \alpha$ ), which implies that  $w$ ,  $y/k$ , and  $k/l$  are all constant in the steady state. Consequently, in view of (T1.6), the  $\dot{c} = 0$  line is linear and negatively sloped.

For a positive birth rate, the  $\dot{c} = 0$  curve is given by the solid line connecting points  $P_1$ ,  $P_2$ , and  $P_3$ . The position and slope of the  $\dot{c} = 0$  line is determined by two effects working in opposite directions: (i) the generational turnover effect; and (ii) the aggregate labor supply

<sup>22</sup>Details of the derivation of the phase diagram are found in Heijdra and Ligthart (2004).

<sup>23</sup>In deriving the equilibrium loci, we take into account that equilibrium employment depends on both  $c$  and  $k$ . Indeed, by combining labor demand (T1.4), labor supply (T1.6), and the production function (T1.3) we find that the labor market equilibrium condition can be written as:

$$(f(l) \equiv) \frac{1-l}{l^{\varepsilon_K}} = \frac{\omega_0 c}{k^{\varepsilon_K}},$$

where  $\omega_0$  is a positive constant,  $f'(\cdot) < 0$ , and  $f''(\cdot) > 0$  (for  $l \in [0, 1]$ ). Given  $k$ , an increase in  $c$  reduces labor supply and thus lowers equilibrium employment.

<sup>24</sup>Strictly speaking,  $\beta = 0$  is not needed to generate  $r = \alpha$  in steady state. See (T.1.2) in Table 2 and the discussion in footnote 16.

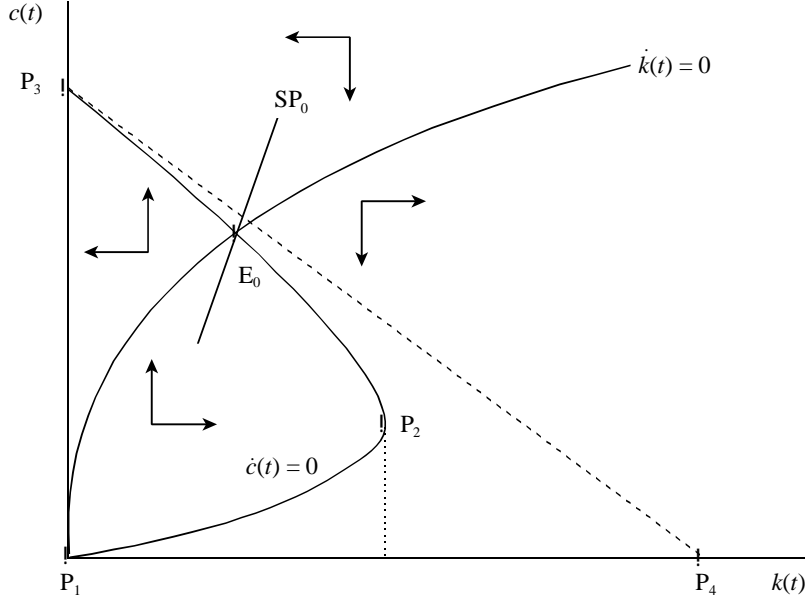


Figure 1: Phase Diagram

effect. The  $\dot{c} = 0$  line is almost horizontal near the origin, where labor supply is close to unity and thus approaches full exogeneity (corresponding to the Blanchard (1985), Buiter (1988) and Weil (1989) models).<sup>25</sup> The  $\dot{c} = 0$  line is upward sloping on the line segment  $P_1P_2$ , reflecting the dominant generational turnover effect. In contrast, on the line segment  $P_2P_3$ , labor supply is fairly elastic, yielding a downward sloping  $\dot{c} = 0$  curve that is steeper than the Ramsey  $\dot{c} = 0$  line (which is given by the dashed line going through points  $P_3$  and  $P_4$ ). If the elasticity of intertemporal labor supply approaches infinity (near  $P_3$ ), the two curves coincide.

The consumption dynamics—illustrated by the vertical arrows in Figure 1—are as follows. For points to the left (right) of the  $\dot{c} = 0$  line, consumption rises (falls) over time. To see why, note the following that the interest rate depends on both  $c$  and  $k$  according to  $r(c, k)$ , where  $\partial r / \partial k < 0$  and  $\partial r / \partial c < 0$ . The  $\dot{c} = 0$  line can thus be written in short-hand notation as  $r(c, k) - \alpha = \eta(\alpha + \beta)\varepsilon_C(k/c)$ . Given  $c$ , a fall (rise) in  $k$  leads to an increase (decrease) in the rate of interest and a decrease (increase) in the  $k/c$  ratio—representing the generational turnover term—yielding an increase (decrease) in consumption growth.

There is a unique equilibrium at point  $E_0$  and the configuration of arrows in Figure 1 confirms that this equilibrium is a saddle point. See the Appendix for a formal proof. The saddle path associated with  $E_0$  is denoted by  $SP_0$ . Although Figure 1 has been drawn under

<sup>25</sup>The  $\dot{c} = 0$  line can only be described parameterically, that is, by varying  $l$  in the feasible interval  $[0, 1]$ . In moving from point  $P_1$  to  $P_3$ ,  $l$  falls from 1 to 0; it follows that in  $P_1$ ,  $l = 1$  and the labor-leisure ratio ( $\omega_{LL}$ ) equals zero, while in  $P_3$ ,  $l \rightarrow 0$  and  $\omega_{LL} \rightarrow \infty$ .

the assumption that the equilibrium occurs along the downward sloping segment  $P_3P_2$  of the  $\dot{c} = 0$  line, it cannot be ruled out *a priori* that the intersection occurs somewhere along the upward sloping segment  $P_1P_2$ . In Section 5, however, we shall argue that the case illustrated in Figure 1 is empirically the most relevant one.

## 4 Qualitative Analysis of Small Demographic Shocks

This section studies the effect of demographic shocks on the optimal savings-labor supply response of the household sector and on the investment decisions of firms. Specifically, we analyze the short-run, transition, and long-run macroeconomic effects of three stylized demographic scenarios, employing the graphical apparatus developed in the previous section. The first shock concerns an exogenous drop in fertility taking as given the mortality rate, which we shall refer to as the *pure baby-bust scenario*. Here, we focus on a drop in the fertility rate rather than a drop in the death rate because the former is in many industrialized countries the more important factor quantitatively. The next two shocks pertain to composite changes as both the fertility and death rates are changed. One scenario is an exogenous decrease in fertility exactly matched by an increase in longevity (that is, a fall in the death rate), so as to maintain a *constant population growth rate*. Although it is a stylized case, some industrialized countries may at times be experiencing this type of shock. Another scenario concerns an exogenous decrease in the birth rate while adjusting the death rate endogenously so as to offset the generational turnover effect of a lower birth rate. It is of interest to analyze the case of *constant generational turnover* because it is not a priori evident whether the macroeconomy would be affected at all. This scenario could be of practical relevance to developing countries at war or to post-conflict economies, where drops in birth rates and rises in death rates often occur simultaneously. It will be shown that the latter two scenarios have qualitatively different macroeconomic results, owing to generational turnover.

To keep matters simple, attention is paid to unanticipated and permanent changes in demographics. The formal proofs underlying the qualitative analysis—obtained by log-linearizing the model around an initial steady state and subsequently perturbing the system—can be found in the Appendix.

### 4.1 Pure Baby Bust

A permanent and unexpected decrease in the fertility rate (that is,  $d\eta < 0$ ), given a constant mortality rate, decreases the population growth rate (that is,  $dn_N = d\eta < 0$ ). As a result, generational turnover decreases; existing generations are replaced by newly born agents at a slower pace.<sup>26</sup> Figure 2 shows the qualitative effects of this so-called pure baby-bust scenario.

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<sup>26</sup>The model features a constant probability of death, implying that a young person has the same expected lifetime—that is, the inverse of the probability of death—as a very old person

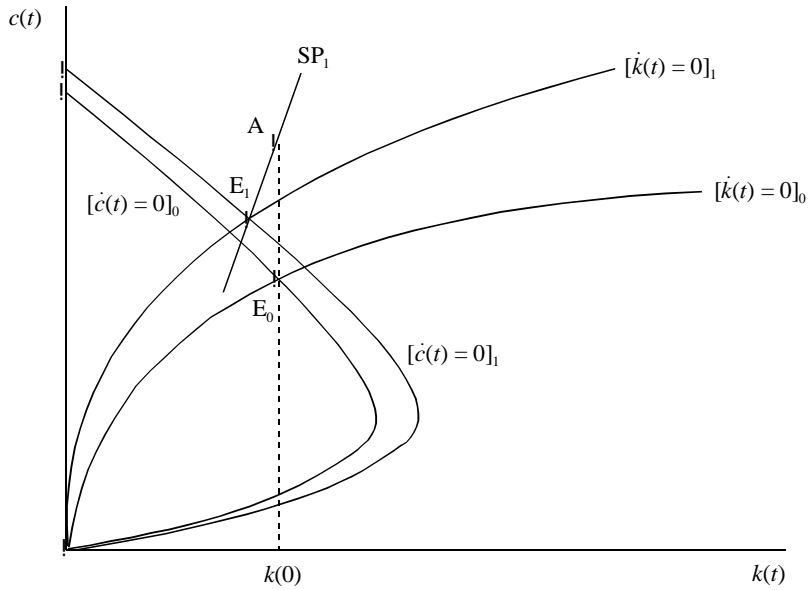


Figure 2: Baby Bust

The sudden drop in fertility shifts the  $\dot{k} = 0$  line up and moves the  $\dot{c} = 0$  line to the right, shifting the long-run equilibrium from point  $E_0$  to  $E_1$ , where per capita consumption has increased. The dashed lines represent the equilibrium loci before the demographic shock, whereas the solid lines represent the loci after the shock. Figure 2 depicts the situation where the shift in the  $\dot{k} = 0$  line is sufficiently large to generate a new equilibrium to the left of the old equilibrium (see the discussion below).

On impact, consumption jumps up to point A on the new saddle path, reflecting the drop in the short-run interest rate, making present consumption more attractive than future consumption. As a result, per capita labor supply (that is, the macroeconomic participation rate) falls, pushing up short-run wages, and thus benefiting young generations—who mainly consume out of wage income—while depressing interest income of the elderly. During transition, consumption gradually falls, mirroring the smooth rise in the interest rate, which, however, remains below its old steady state value.

Per capita consumption is higher<sup>27</sup> and per capita labor supply is lower in the new long-run equilibrium. The long-run effect on the per capita capital stock (that is, the capital intensity) is ambiguous; it depends on whether the generational turnover effect is dominated by the labor supply effect. If individual consumption growth profiles are fairly flat—and thus the generational turnover effect is weak—and intertemporal labor supply is sufficiently elastic, the optimal per capita capital stock falls.<sup>28</sup> In view of the smaller population increment, *aggregate*

<sup>27</sup>This partly reflects what Cutler and others (1990) have labeled the “Solow effect.” Due to the reduction in the population growth rate a lower amount of savings is required to maintain a given per capita capital stock.

<sup>28</sup>This requires that the initial birth and death rates are small so that the rightward shift in the  $\dot{c} = 0$  line

savings must have fallen by more than *per capita* savings. In the long run, the participation rate is below its old steady-state level, reflecting reduced labor supply (as consumption is higher) and lower labor demand associated with lower per capita assets in the production sector. On a net basis, long-run wages have risen and—from the factor price frontier—interest rates have fallen compared to the old steady state. Accordingly, young agents consuming out of human capital benefit, while the elderly lose out. Table 3 compares the impact and long-run effects on the per capita macroeconomic variables.

If individual consumption growth profiles are relatively steep, the per capita capital stock may even increase. Intuitively, the positive savings effect associated with the reduction in generational turnover attenuates the fall in aggregate savings induced by the lower rate of interest. Overlapping generations may thus give rise to diametrically different results than those derived in the infinite-horizon Ramsey model. The assumptions made on the elasticity of labor supply are crucial in this respect.<sup>29</sup> If labor supply is exogenous, the per capita capital stock unambiguously rises as is also shown in the life-cycle model by Elmendorf and Sheiner (2000).<sup>30</sup>

The Appendix shows that the results of Cutler and others (1990), who assume infinitely-lived agents, exogenous population growth and exogenous labor supply, are a special case of our model. In their framework, a fertility drop yields a reduction in *aggregate* steady-state savings, whereas the optimal capital intensity remains—in contrast to our results—unaffected.<sup>31</sup> Allowing for endogenous labor supply reinforces this negative savings effect, yielding a reduction in the optimal capital intensity.

## 4.2 Stationary Population Growth

Consider a demographic change which involves simultaneously decreasing the birth rate and mortality rate (that is,  $d\eta = d\beta < 0$  and thus  $dn_N = 0$ ) so as to yield a stationary population growth rate. Generational turnover falls, but by less than in the pure baby-bust scenario. Figure 3 shows that the shock leaves the  $\dot{k} = 0$  line unaffected, but shifts the  $\dot{c} = 0$  line to the right, yielding a higher capital intensity. Symmetric changes in the birth and death rates thus have a non-neutral effect on the economy.

On impact, the rate of interest rises, making current consumption less attractive compared to future consumption. The latter is represented by a downward jump in consumption from point  $E_0$  to point A on the new saddle path  $SP_1$ . The fall in consumption per capita shifts

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is sufficiently small. With initial birth and death rates close zero—that is,  $r \approx \alpha$ , thereby approximating the case of infinitely-lived households—the per capita capital stock would unambiguously fall.

<sup>29</sup>In the following, it is assumed that the labor supply effect is sufficiently strong to generate a stable equilibrium on the downward-sloping section of the consumption equilibrium locus (Figure 1).

<sup>30</sup>Elmendorf and Sheiner (2000) employ a Diamond model with exogenous population growth, but they do not work out the comparative statics analytically.

<sup>31</sup>Cutler and others (1990) thus effectively set  $\beta < 0$  and  $\eta = 0$  (see footnote 16), so that  $r(k) = \alpha$ , explaining why the optimal capital intensity remains unaffected in the steady state.



**Table 3. Summary of the Qualitative Effects of Various Demographic Shocks<sup>(§)</sup>**

<i>Demographic Shock</i>	<i>Period</i>	<i>y</i>	<i>k</i>	<i>l</i>	<i>c</i>	<i>w</i>	<i>r</i>
Pure Baby Bust	impact	-	0	-	+	+	-
	long run	-	-	-	+	+	-
Stationary Population Growth	impact	+	0	+	-	-	+
	long run	+	+	+	+	+	-
Constant Generational Turnover	impact	-	0	-	+	+	-
	long run	-	-	-	+	+	-

(§) It is assumed that the generational turnover effect is dominated by the labor supply effect.

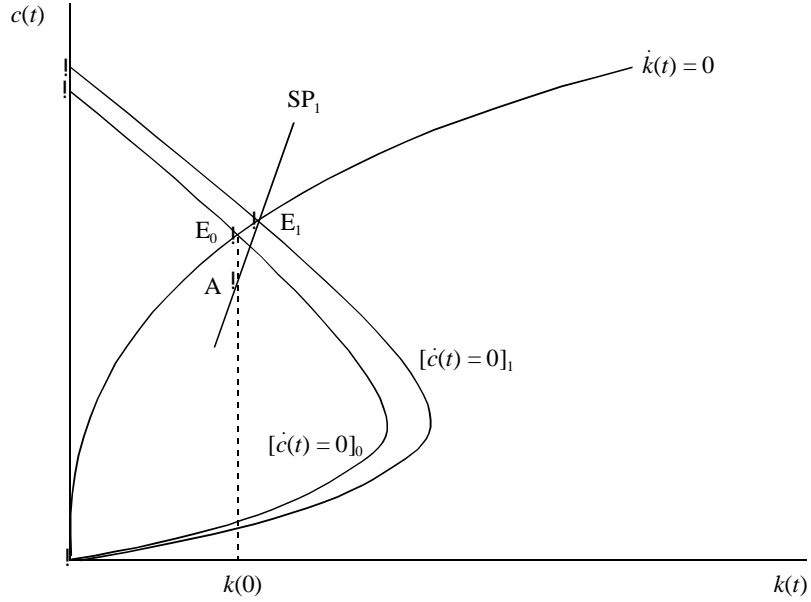


Figure 3: Stationary Populations Growth

the short-run labor supply curve to the right, so that for a given level of labor demand, per capita employment and per capita output rise and wages fall. The simultaneous decrease in per capita consumption and increase in per capita output crowds in investment during transition.

Consumption gradually increases along the transition path to the new steady state, where per capita capital accumulation and per capita consumption are higher than in the old steady state. Given the stationarity of the population growth rate, the *aggregate* stock of capital and *aggregate* consumption have increased as well. The rise in the capital stock increases labor demand, but the rise in consumption induces households to supply less labor. On a net basis, equilibrium employment rises. The steady-state interest rate falls and the wage rate rises, reflecting an increase in the long-run capital-labor ratio.

The qualitative effects of a decline in population turnover are identical to those of a capital subsidy (see Appendix).<sup>32</sup> Both per capita output and per capita savings increase. Due to the decline in generational turnover, newly born generations—not owning any financial capital yet—are added to the population at a slower pace, which raises the average asset holdings of the population. The subsidy increases average asset holdings by raising the return to savings.

### 4.3 Constant Generational Turnover

Rather than keeping the population growth rate constant, one could also consider a demographic change which leaves unaffected the rate of generational turnover (that is, the second term in (T1.2)). This requires that the death rate has to rise by  $d\beta = -(\alpha + \beta)(d\eta/\eta) > 0$  to compensate for the fall in population turnover induced by the reduced rate at which new generations are born (that is,  $d\eta < 0$ ). Accordingly,  $dn_N = (\alpha + \beta + \eta)(d\eta/\eta) < 0$ , implying a reduction in net population growth, which is larger than under a pure baby bust scenario.

The effects of the shock can be analyzed with the aid of Figure 4. The  $\dot{c} = 0$  line remains unaffected, but the  $\dot{k} = 0$  shifts up. The short-run and long-run qualitative effects are equivalent to those of a pure baby bust. In the new steady state, per capita consumption has increased while the per capita capital stock has fallen. Quantitatively, the two demographic shocks do differ, however. Section 5 shows that the fall in the per capita capital stock is larger under constant generational turnover for reasonable values of the parameters.

As is shown in the Appendix, the demographic scenario of constant generational turnover yields macroeconomic results qualitatively similar to a fall in per capita public spending. Intuitively, both decrease labor market participation via the income effect in aggregate labor supply, thereby raising wages and reducing long-run rates of interest. The latter discourages household savings.

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<sup>32</sup>See Heijdra and Ligthart (2000) for an overview of the macroeconomic effects of capital income taxes.

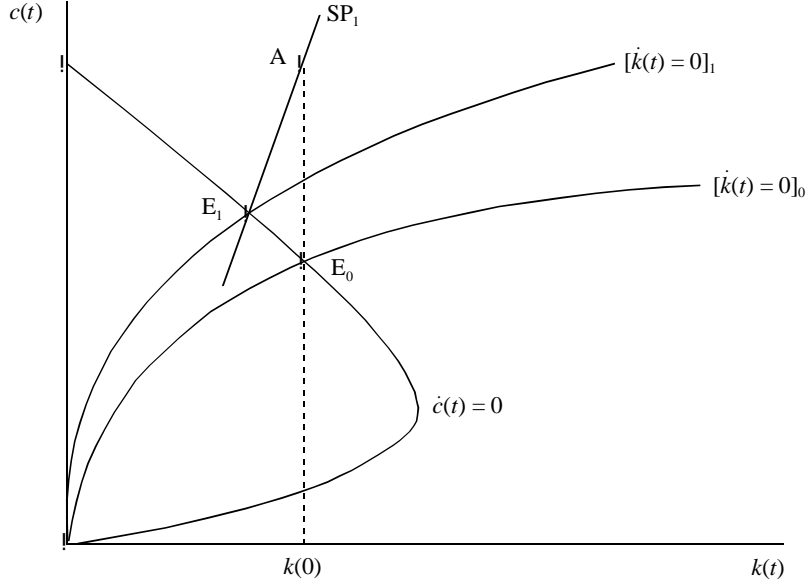


Figure 4: Constant Generational Turnover

## 5 Quantitative Analysis of Non-Infinitesimal Demographic Shocks

This section illustrates the quantitative significance of demographic shocks for two reasons. First, we would like to assess whether the effect of demographic shocks on the macroeconomic variables is of first-order or second-order nature. If it is true that the effects are of second order nature we have to worry less about demographic changes from a policy point of view. Furthermore, we would like to know more about the significance of the factor time in this respect. Are demographic changes necessarily long-run phenomena, which generally yield larger output responses in the long run than in the short run? Finally, we would like to get a feel for the sensitivity of the results to changes in key parameters (that is, the pure rate of time preference ( $\alpha$ ), death rate ( $\beta$ ), and birth rate ( $\eta$ )).

Second, we study what kind of fiscal policy response should be devised to counteract the macroeconomic effects of demographic shocks. Since public debt has been abstracted from, the analysis focuses on “above-the-line” measures such as tax and expenditure policies.<sup>33</sup> In particular, we are interested in knowing what kind of fiscal policy is needed to deal with the effects of a decline in the fertility rate. It allows us to answer the question whether a standard fiscal policy is applicable in all cases. Note that the chosen objective of neutralizing the effects of demographic shocks is not meant to describe actual or desired government behavior as we have not specified the government’s objective function formally. We have taken this approach

<sup>33</sup>Note that consumption and labor taxes have been abstracted from.

to illustrate the main argument without complicating the modeling exercise too much.

## 5.1 Calibration

A discrete time version of the continuous time model—as set out in Table 2—is calibrated for plausible values of the parameters. Rather than employing a log-linear approximation, as is customary in the literature, the model is simulated in non-linear format, allowing us to study non-infinitesimal shocks. The DYNARE software—running under MATLAB—is employed for this purpose. See Juillard (1996). The variables are written in stationary format, where flow variables at time  $t$  are expressed in terms of the population stock measured at end of period. In the simulations, the economy is given 40 years to reach a new steady-state after a shock, which is an arbitrary number, but is sufficient in the present set up.

The benchmark model is calibrated as follows. We have set the instantaneous probability of death at 1 percent a year, implying a life expectancy of a 100 years. The birth rate is taken at 1.5 percent, resulting in positive population growth in the benchmark scenario. This is closely in line with the data. Crude death rates for developed regions amounted to 1.02 percent per 100 of the population during 1990-1995, whereas birth rates were 1.4 percent per 100 of the population during that same time period (see United Nations (2003)). The pure rate of time preference is assumed to be 4 percent, the capital income share ( $\varepsilon_K$ ) is set to 0.35, the consumption share in utility ( $\varepsilon_C$ ) is 0.32, and the rate of capital depreciation amounts 6 percent. Initially, the capital tax rate is set to zero. These parameter values guarantee that the economy is operating on the downward-sloping section of the  $\dot{c} = 0$  line, which is the economically meaningful equilibrium. Once the values of the basic parameters are set, all the information on the other variables and parameters can be derived.<sup>34</sup>

## 5.2 Pure Baby Bust

Table 4 shows the quantitative allocation effects of a decline in the birth rate at time  $t = 0$  by 10 percent for various values of the key demographic parameters. For example, in the benchmark scenario,  $\eta$  changes from  $\eta_0 = 0.0150$  to  $\eta_1 = 0.0135$ . The reported values in the table are elasticities. For example, the output effect is given by  $(dy/y)/(d\eta/\eta) \times 100$ . The elasticities are calculated at the end of the first period (which we refer to as the “short-run” and is identified by  $t = 1$ ), during transition (where an arbitrary point  $t = 10$  is taken) and in the new steady state (labeled by  $t \rightarrow \infty$ ). All the output elasticities are well above unity, suggesting that the effects of demographic changes are of first-order nature.

Panel (a) of Table 4 reports the numerical effects for various values of the death rate. It shows that per capita capital stock declines and per capita consumption increases in the new equilibrium (see the column labeled  $t \rightarrow \infty$ ). Steady-state output and employment decline.

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<sup>34</sup>These values correspond to  $\omega_{LL} = 2$ , which denotes the intertemporal substitution elasticity of labor supply (see equation (A.5) in the Appendix).

**Table 4. Allocation Effects of a Pure Baby Bust for Various Values of Key Parameters (In Percent)<sup>(§)</sup>**

(a)	$\beta = 0$			$\beta = 0.015$			$\beta = 0.03$		
	$t = 1$	$t = 10$	$t \rightarrow \infty$	$t = 1$	$t = 10$	$t \rightarrow \infty$	$t = 1$	$t = 10$	$t \rightarrow \infty$
$y$	-3.58	-3.84	-4.02	-2.89	-3.06	-3.11	-3.16	-3.31	-3.35
$k$	-0.44	-1.68	-2.73	-0.19	-1.15	-1.53	-0.19	-1.12	-1.48
$l$	-5.50	-5.09	-4.70	-4.44	-4.12	-3.95	-4.86	-4.53	-4.36
$c$	4.84	3.94	3.17	3.74	3.09	2.79	3.94	3.29	3.00
$w$	1.93	1.25	0.69	1.56	1.06	0.85	1.71	1.22	1.01
$r$	-8.78	-5.71	-3.13	-7.06	-4.83	-3.85	-7.70	-5.49	-5.54
(b)	$\alpha = 0.01$			$\alpha = 0.02$			$\alpha = 0.04^{(\ddagger)}$		
	$t = 1$	$t = 10$	$t \rightarrow \infty$	$t = 1$	$t = 10$	$t \rightarrow \infty$	$t = 1$	$t = 10$	$t \rightarrow \infty$
$y$	-5.47	-5.98	-6.07	-4.56	-4.95	-5.05	-3.42	-3.68	-3.76
$k$	-0.44	-3.01	-4.43	-0.37	-2.42	-3.45	-0.28	-1.69	-2.26
$l$	-8.40	-7.76	-6.94	-7.00	-6.38	-5.90	-5.26	-4.80	-4.56
$c$	7.84	6.16	4.90	6.28	4.93	4.08	4.49	3.53	3.10
$w$	2.95	1.71	0.87	2.46	1.44	0.86	1.85	1.12	0.81
$r$	-35.45	-20.49	-10.42	-17.49	-10.28	-6.11	-8.37	-5.09	-3.66
(c)	$\eta = 0.01$			$\eta = 0.02$			$\eta = 0.04$		
	$t = 1$	$t = 10$	$t \rightarrow \infty$	$t = 1$	$t = 10$	$t \rightarrow \infty$	$t = 1$	$t = 10$	$t \rightarrow \infty$
$y$	-2.29	-2.46	-2.51	-4.55	-4.89	-5.00	-8.92	-9.60	-9.83
$k$	-0.19	-1.15	-1.54	-0.37	-2.21	-2.95	-0.67	-3.96	-5.27
$l$	-3.52	-3.20	-3.03	-6.98	-6.40	-6.10	-13.96	-12.74	-12.27
$c$	2.96	2.31	2.01	6.06	4.81	4.24	12.62	10.40	9.43
$w$	1.23	0.74	0.52	2.46	1.51	1.10	4.84	3.17	2.47
$r$	-5.64	-3.38	-2.39	-11.05	-6.82	-4.98	-21.02	-13.84	-10.79

(§) A decline in the birth rate by 10 percent, which implies changing  $\eta$  in the benchmark scenario from  $\eta_0 = 0.015$  to  $\eta_1 = 0.0135$ . All values in the table are elasticities, for example, per capita output changes according to  $(dy/y)(d\eta/\eta) \times 100$ . ( $\ddagger$ ) identifies the benchmark scenario.

The fall in the per capita capital stock is smaller for larger values of the death rate (that is, households face a shorter planning horizon). Intuitively, households internalize less of the fall in the capital stock as the future is discounted more heavily. Note that aggregate employment per capita, private consumption per capita and the price variables (that is, the rate of interest and wage rate) respond more strongly to the policy change in the short run than in the long run, whereas the drop in per capita output is larger in the long run (compare columns  $t = 1$  and  $t \rightarrow \infty$ ). Obviously, capital is a sluggish variable and will therefore not adjust much in the first period, while at impact it does not change at all.

Panel (b) of Table 4 shows that the fall in the per capita capital stock, per capita output and labor market participation is larger for smaller values of the pure rate of time preference (implying that households are more patient). In that case, the future is discounted less heavily, causing agents to internalize more of the effects of the shock. Moreover, the difference between the steady-state elasticities and impact elasticities is larger for a smaller rate of time preference.

What happens to the macroeconomic variables if generations are more disconnected, or in other words, if initial birth rates are higher at the time of the drop in the fertility rate? Panel (c) of Table 4 shows output, capital and employment in the new steady state fall by more for larger values of the birth rate. Consumption overshoots its new equilibrium initially, particularly if initial birth rates are large. In addition, the interest rate elasticity exceeds the output elasticity at high birth rates, while the opposite is observed for low birth rates. Note that the rate of interest rates is generally more sensitive to demographic shocks than the wage rate.

### 5.3 Composite Demographic Shocks

The top panel of Table 5 shows that a decrease in the birth and mortality rates so as to maintain a stationary population growth rate (labeled by SPG) causes an increase in steady-state output per capita, the per capita capital stock and consumption per capita, in line with the results of Section 4. The steady-state capital stock per capita responds more strongly to the shock than steady-state output per capita, owing to the small response in the macroeconomic participation rate. Per capita consumption initially falls, and quickly recovers during transition (see the column labeled  $t = 10$ ) to yield a higher per capita consumption level in the new steady state. The macroeconomic labor market participation rate rises on impact, but its increase is smaller in the new steady state due to the rise in steady-state consumption. The latter generates a negative income effect in labor supply.

The simulation results for a drop in the birth rate and rise in the death rate yielding constant generational turnover (CGT) are set out in the bottom panel of Table 5. A larger probability of death leads to larger per capita capital losses. Comparing the results with those of the pure baby bust scenario, shows that the steady-state elasticities of the per capita capital

**Table 5. Allocation Effects of Composite Demographic Shocks for Various Values of the Death Rate (In Percent)<sup>(§)</sup>**

		$\beta = 0.005$			$\beta = 0.015$			$\beta = 0.030$		
		$t = 1$	$t = 10$	$t \rightarrow \infty$	$t = 1$	$t = 10$	$t \rightarrow \infty$	$t = 1$	$t = 10$	$t \rightarrow \infty$
SPG	$y$	0.66	0.98	1.09	0.70	1.03	1.13	0.75	1.08	1.16
	$k$	0.34	2.03	2.71	0.37	2.20	2.93	0.40	2.41	3.20
	$l$	1.01	0.48	0.21	1.08	0.47	0.16	1.15	0.43	0.06
	$c$	-0.88	0.25	0.77	-0.91	0.33	0.90	-0.93	0.45	1.07
	$w$	-0.35	0.50	0.88	-0.38	0.56	0.97	-0.40	0.65	1.10
	$r$	1.62	-2.28	-3.99	1.72	-2.56	-4.91	1.82	-2.93	-4.97
CGT	$y$	-1.58	-1.79	-1.85	-1.80	-2.01	-2.07	-2.12	-2.33	-2.38
	$k$	-0.22	-1.33	-1.77	-0.25	-1.49	-1.98	-0.29	-1.72	-2.27
	$l$	-2.42	-2.07	-1.88	-2.76	-2.34	-2.12	-3.24	-2.71	-2.44
	$c$	2.12	1.36	1.01	2.35	1.48	1.08	2.66	1.64	1.18
	$w$	0.86	0.29	0.04	0.98	0.33	0.05	1.16	0.39	0.06
	$r$	-3.88	-1.31	-0.17	-4.41	-1.49	-0.21	-5.15	-1.75	-0.26

(§) A decline in the birth rate by 10 percent, which implies changing  $\eta$  from  $\eta_0 = 0.015$  to  $\eta_1 = 0.0135$ . All values in the table are elasticities, for example, per capita output changes according to  $(dy/y)(d\eta/\eta) \times 100$ .

stock and output are larger under CGT, reflecting the higher rate of generational turnover in the latter case. Intuitively, more capital is lost due to generations passing away. Given that capital and labor are cooperative factors of production, employment and labor market participation increase in the new steady state. Private consumption and output, however, are negatively correlated. Note that the aggregate participation rate is the main shock absorber in the new steady state, whereas “prices” do not change much.

## 5.4 Stabilization Policy

We now turn to the question of what kind of stabilization policy can be employed to counteract the effects of the three demographic shocks that were analyzed. It should be emphasized that we do not intend to perform an exhaustive comparative analysis of instruments, but just want to illustrate what kind of fiscal measures—given the available set of fiscal instruments—could be implemented to deal with the implications of demographic shocks.

Exactly offsetting the steady-state effects of a pure baby bust (PBB) would require a rise in the capital tax to shift the  $\dot{c} = 0$  line to the left and a rise in public spending to move the  $\dot{k} = 0$  line down. The required tax and expenditure policy mix for the continuous time model

follows from (A.10) in the Appendix:

$$d\tau = - \left( \frac{(1-\tau)(r-\alpha)}{r+\delta} \right) \frac{d\eta}{\eta} > 0, \quad \frac{dg}{g} = - \left( \frac{k\eta}{y\omega_G} \right) \frac{d\eta}{\eta} > 0, \quad (19)$$

where  $\omega_G$  denotes the share of government consumption in real output.

Panel (a) of Table 6 provides a numerical example for various values of the death rate.<sup>35</sup> Because all effects on the macroeconomic variables are zero we do not have to report them. In all cases, net public spending—that is, total expenditures minus capital taxes—has to rise to compensate for the output loss under the baby bust. Accordingly, households face higher lump-sum taxes which are used to finance the fiscal impulse. If death rates rise, we saw that output losses are smaller (Table 4) so that a smaller per capita level of public spending is needed to counteract the shock. Hence, a smaller increase in lump-sum taxes results.

As was argued in Section 4, capital subsidies have qualitatively similar output effects to those of a demographic shock that keeps the population growth rate constant. Capital taxes can thus be employed to counteract the steady-state output effect of an SPG scenario. The required change in capital taxes is given by:

$$\tau_\infty = \tau_0 - \left( \frac{(r-\alpha)(1-\tau_0)(\alpha+\beta+\eta)}{(r+\delta)(\alpha+\beta)} \right) \frac{d\eta}{\eta} > 0, \quad (20)$$

where  $\tau_0$  and  $\tau_\infty$  are the capital taxes in the old and new steady state, respectively. Panel (b) of Table 6 shows that the calculated capital tax rates are quite small, ranging from 0.17 percent to 0.21 percent, which is not surprising given that the output elasticities are close to unity. At higher values of the death rate, a larger capital tax rate is needed to offset the demographic change, permitting lower lump-sum taxes on the household sector.

Panel (c) of Table 6 analyzes what change in per capita government consumption is needed to counteract the macroeconomic effects of a CGT scenario. Analytically, to leave the economy unaffected, it is required that:

$$\frac{dg}{g} = - \left( \frac{k}{y} \right) \left( \frac{\alpha+\beta+\eta}{\omega_G} \right) \frac{d\eta}{\eta} > 0, \quad (21)$$

which follows from (A.10) of the continuous time model set out in the Appendix. As is evident, a less restrictive fiscal policy is needed, particularly if death rates are large. Intuitively, at higher values of  $\beta$  the output elasticity is smaller than at low values, necessitating a larger boost in public spending to offset the effects of a given drop in fertility.

## 6 Concluding Remarks

The paper studies the dynamic macroeconomic effects of demographic shocks employing a Yaari-Blanchard overlapping generations framework extended for endogenous labor supply

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<sup>35</sup>The values obtained from (19) are used as initial guesses in the non-linear simulations to pin down the required mix.



**Table 6. Long-Run Stabilization Policy (In Percent)<sup>(§)</sup>**

<i>Demographic Scenario</i>	<i>Instrument</i>	$\beta = 0.005$	$\beta = 0.015$	$\beta = 0.03$
(a) PBB	$dg/g$	2.72	2.64	2.50
	$d\tau/(1 - \tau)$	0.1210	0.1497	0.1790
	$dz/z$	2.25	1.96	1.80
(b) SPG	$d\tau/(1 - \tau)$	0.1685	0.1870	0.2110
	$dz/z$	-1.05	-1.13	-1.22
(c) CGT	$dg/g$	12.14	13.76	15.89
	$dz/z$	12.14	13.76	15.89

(§) All values are percentage deviations from the old steady state.

while allowing for a richer demography. The model encompasses both finitely-lived and infinitely-lived households, thereby facilitating a comparison with results from the existing literature. Besides analytical results, numerical examples are used to analyze the sensitivity of the macroeconomy to various demographic shocks.

The main results can be summarized as follows. With overlapping generations, a drop in fertility does not necessarily lead to a reduction in per capita savings and output as is derived in the standard infinitely-lived household model. Per capita savings may increase if the effect of generational turnover is sufficiently strong to dominate the aggregate labor supply effect, but this is not the empirically relevant case.

Endogenizing labor supply in a model with positive population growth and infinitely-lived households reinforces the negative aggregate savings effect found by Cutler and others (1990). With endogenous labor supply, aggregate savings fall by more than under exogenous labor supply, giving rise to a fall in per capita savings and per capita output. With exogenous labor supply, however, the capital intensity is left unaffected by the shock.

Depending on the nature of the demographic change, the steady state effects on the macroeconomy differ. A pure baby bust gives rise to a fall in steady-state output, but a rise in per capita consumption. A drop in fertility while keeping generational turnover constant by adjusting the death rate yields results qualitatively similar to those of a pure baby bust. The qualitative effects of a decline in generational turnover at constant population growth are diametrically different. Output per capita increases, but steady-state consumption per capita declines. Policy makers should therefore carefully analyze what changes in demography give rise to population dynamics before prescribing a suitable long-run fiscal policy response. Under a scenario of constant generational turnover a restrictive expenditure policy would counteract the effects of the shock, against the needed increase in capital taxes under a scenario of a stationary population growth rate.

The short-run effects of demographic changes can differ markedly from the long-run effects,

not only quantitatively, but also qualitatively. For example, a proportional decline in death and fertility rates yields a decline in per capita consumption on impact, but increases per capita consumption in the new steady state. A baby bust scenario, however, gives rise to overshooting in per capita consumption while yielding higher per capita consumption in the new steady state. Notice that long-run wages are pushed up in all three scenarios, and generally rise in the short run too, except in a demographic scenario that keeps the population growth rate constant.

Numerical results show that the macroeconomic elasticities of demographic shocks are well above unity for reasonable values of the parameters, suggesting that demographic changes have significant effects on the macroeconomy. Not surprisingly, the output elasticity is generally larger in the long run than in the short-run, which is not necessarily the case for the macroeconomic control variables. In addition, steady-state interest rates seem to be more sensitive to shocks than wages.

The analysis has abstracted from government debt and labor income taxes as means to offset the effects of demographic shocks. Furthermore, we have not looked at the welfare effects of demographic changes. This would be particularly relevant in the study of the design of optimal policies to address population aging. This is left for further research.

## Appendix: Model Solution

### A.1 General Solution Approach

In this appendix we show how the main results mentioned in the text were derived. We log-linearize the model of Table 1 around an initial steady state, using the notational conventions mentioned at the beginning of Section 2. After some simplifications, the following quasi-reduced form equations can be derived:

$$\tilde{y}(t) = \phi \varepsilon_K \tilde{k}(t) - (\phi - 1) \tilde{c}(t), \quad (1)$$

$$\omega_I \tilde{i}(t) = \tilde{y}(t) - \omega_C \tilde{c}(t) - \omega_G \tilde{g}(t), \quad (2)$$

$$(1 - \varepsilon_K) \tilde{l}(t) = \tilde{y}(t) - \varepsilon_K \tilde{k}(t), \quad (3)$$

$$-\varepsilon_K [\tilde{y}(t) - \tilde{k}(t)] = (1 - \varepsilon_K) \tilde{w}(t) = -\varepsilon_K \left[ \left( \frac{r}{r + \delta} \right) \tilde{r}(t) + \tilde{\tau}(t) \right], \quad (4)$$

where a tilde ( $\sim$ ) denotes a relative change (for example,  $\tilde{y}(t) \equiv dy/y$ , except for  $\tilde{\tau} \equiv d\tau/(1 - \tau)$ ),  $\omega_C \equiv c/y$  denotes the share of private consumption in real output,  $\omega_I \equiv i/y$  is the share of investment in real output, and  $\omega_G \equiv g/y$  denotes the share of government consumption in real output.

The parameter  $\phi$  represents the intertemporal labor supply effect, which is defined as:

$$\phi \equiv \frac{1 + \omega_{LL}}{1 + \omega_{LL} \varepsilon_K} \geq 1, \quad (5)$$

where  $\omega_{LL}$  ( $\equiv (1-l)/l = (N-L)/L \geq 0$ ) is the ratio of leisure to labor, which also represents the *aggregate* intertemporal substitution elasticity of labor supply. Notice that  $\phi = 1$  if labor supply is exogenous (because  $l = 1$  or  $N = L$  implies that  $\omega_{LL} = 0$ ). Since  $\omega_{LL} \geq 0$  the sign restriction on  $\phi$  is automatically satisfied if  $\varepsilon_K \geq 0$ . If  $\varepsilon_K > 0$ ,  $\phi$  is a concave function of  $\omega_{LL}$  with a positive asymptote of  $1/\varepsilon_K$  as  $\omega_{LL} \rightarrow \infty$ , and if  $\varepsilon_K = 0$ , we arrive at  $\phi = 1 + \omega_{LL} \geq 1$ .

The dynamics of the per capita capital stock and per capita consumption are given by:

$$\dot{\tilde{k}}(t) = \omega_I \left( \frac{y}{k} \right) [\tilde{i}(t) - \tilde{k}(t)] - \eta\tilde{\eta} + \beta\tilde{\beta}, \quad (6)$$

$$\dot{\tilde{c}}(t) = (r - \alpha) \left[ \tilde{c}(t) - \tilde{k}(t) - \tilde{\eta} - \left( \frac{\beta}{\alpha + \beta} \right) \tilde{\beta} \right] + r\tilde{r}(t), \quad (7)$$

where a variable with a tilde and a dot is the time rate of change (relative to the initial steady state) and  $y/k = (r + \delta)/(\varepsilon_K(1 - \tau))$ . Using equations (1)-(4) and (6)-(7) the model can be reduced to a two-dimensional system of first-order differential equations in the per capita capital stock,  $\tilde{k}(t)$ , and per capita private consumption,  $\tilde{c}(t)$ . In its most general form, the dynamic system can be written as:

$$\begin{bmatrix} \dot{\tilde{k}}(t) \\ \dot{\tilde{c}}(t) \end{bmatrix} = \Delta \begin{bmatrix} \tilde{k}(t) \\ \tilde{c}(t) \end{bmatrix} - \begin{bmatrix} \gamma_K(t) \\ \gamma_C(t) \end{bmatrix}, \quad (8)$$

where  $\Delta$  denotes the Jacobian matrix (with typical element  $\delta_{ij}$ , where  $i, j = 1, 2$ ) evaluated at steady state:

$$\Delta \equiv \begin{bmatrix} \frac{y}{k}(\phi\varepsilon_K - \omega_I) & -\frac{y}{k}(\omega_C + \phi - 1) \\ -[(r - \alpha) + (r + \delta)(1 - \phi\varepsilon_K)] & (r - \alpha) - (r + \delta)(\phi - 1) \end{bmatrix}, \quad (9)$$

and  $\gamma_K(t)$  and  $\gamma_C(t)$  are shock terms:

$$\begin{bmatrix} \gamma_K(t) \\ \gamma_C(t) \end{bmatrix} = \begin{bmatrix} \left( \frac{y}{k} \right) \omega_G \tilde{g} + d\eta - d\beta \\ (r - \alpha) \left( \frac{d\eta}{\eta} + \frac{d\beta}{\alpha + \beta} \right) + (r + \delta)\tilde{\tau} \end{bmatrix}, \quad (10)$$

where it can be shown that the determinant of the Jacobian matrix is negative, that is,

$$\begin{aligned} |\Delta| &= -\lambda_1 \lambda_2 \\ &= -(r + \delta) \left( \frac{y}{k} \right) \left[ \left( \frac{r - \alpha}{r + \delta} \right) (\phi(1 - \varepsilon_K) - \omega_G) + (\phi - 1)\omega_G + \omega_C(1 - \phi\varepsilon_K) \right] < 0 \end{aligned} \quad (11)$$

where  $\phi(1 - \varepsilon_K) - \omega_G > 0$ ,  $-\lambda_1 < 0$  is the stable characteristic root, and  $\lambda_2 > 0$  is the unstable root. The latter satisfies the inequality  $\lambda_2 > r - \alpha + \omega_C(r + \delta)$ , which we employ to sign the short run consumption change. Thus, there exists a unique steady state.

The Laplace transform<sup>36</sup> method, as employed in Judd (1982), is used here to solve the model (see Heijdra and Ligthart (2004)). This yields the following long-run effects:

$$\begin{bmatrix} \dot{\tilde{k}}(\infty) \\ \dot{\tilde{c}}(\infty) \end{bmatrix} = \frac{\text{adj}(\Delta)}{|\Delta|} \begin{bmatrix} \gamma_K \\ \gamma_C \end{bmatrix}, \quad (12)$$

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<sup>36</sup>The Laplace transform of  $x(t)$  is denoted by  $\mathcal{L}\{x, s\} \equiv \int_0^\infty x(t)e^{-st}dt$ . Intuitively  $\mathcal{L}\{x, s\}$  represents the present value of  $x(t)$  using  $s$  as the discount rate.

where  $t = \infty$  identifies the steady state after the shock and  $\text{adj}(\Delta)$  is the adjoint matrix of  $\Delta$ .

## A.2 Comparative Statics

### A.2.1 Pure Baby Bust

Let us first consider the pure baby bust scenario. Using (10) and (12) we can derive the long-run effects of an exogenous decrease in the fertility rate (that is,  $d\eta < 0$ ) while keeping the death rate constant (that is,  $d\beta = 0$ ). The effect on the steady-state per capita capital stock is ambiguous:

$$\tilde{k}(\infty) = \left[ (r - \alpha) \left( 1 + \frac{\omega_C}{\eta} \left( \frac{y}{k} \right) \right) + \left( \frac{r - \alpha}{\eta} \right) \left( \frac{y}{k} \right) (\phi - 1) - (r + \delta)(\phi - 1) \right] |\Delta|^{-1} d\eta \leq 0, \quad (13)$$

where  $r > \alpha$  due to the overlapping generations structure of the model and  $|\Delta|^{-1} d\eta > 0$ . If the initial death rate is small—and thus individual consumption growth profiles are fairly flat—and intertemporal labor supply is sufficiently elastic, a drop in fertility depresses the per capita capital stock.

In the infinite horizon Ramsey model, featuring  $r = \alpha$  in steady state, the first and second term of (13) drop out,<sup>37</sup> giving rise to an unambiguous decline in *per capita* savings and thus a smaller per capita capital stock. If, in addition, labor supply is exogenous (that is,  $\phi = 1$ ), the second term drops out as well, yielding the familiar Ramsey result of a constant capital intensity. Accordingly, *aggregate* savings decline as is also shown by Cutler and others (1990).

With overlapping generations (that is,  $r > \alpha$ ) and exogenous labor supply (that is,  $\phi = 1$ , so that the second and third term of (13) drop out), the per capita capital stock unambiguously rises:

$$\tilde{k}(\infty) = (r - \alpha) \left[ 1 + \left( \frac{1}{\eta} \right) \left( \frac{y}{k} \right) \omega_C \right] |\Delta|^{-1} d\eta > 0. \quad (14)$$

Making use of (10) and (12) again, we can derive the long-run effect of a baby bust on per capita consumption:

$$\tilde{c}(\infty) = \left[ (r - \alpha) \left( 1 + \left( \frac{1}{\eta} \right) \left( \frac{y}{k} \right) (\phi \varepsilon_K - \omega_I) \right) + (r + \delta)(1 - \phi \varepsilon_K) \right] |\Delta|^{-1} d\eta > 0, \quad (15)$$

which is unambiguously positive. Initial per capita consumption changes according to:

$$\tilde{c}(0) = \frac{[(r - \alpha)\delta_{12} + (\lambda_2 - \delta_{22})] d\eta}{\eta \delta_{12} \lambda_2} \leq 0, \quad (16)$$

where  $t = 0$  identifies the time of the shock. The initial effect on consumption is positive if the labor supply effect dominates the generational turnover effect. Using equations (1)-(4)

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<sup>37</sup>Note that  $r = \alpha$  for  $\eta = 0$  because there is no extra-dynasty growth.

together with (12) and (15), the steady-state and impact effects on  $l, i, r$ , and  $w$  can be derived as well.<sup>38</sup>

### A.2.2 Stationary Population Growth Rate

The stationary population growth rate scenario implies an equiproportionate fall in the birth and death rate (that is,  $d\eta = d\beta < 0$ , so that  $dn_N = 0$ ). The per capita capital stock rises in the new steady state:

$$\tilde{k}(\infty) = \frac{(r - \alpha)(\alpha + \beta + \eta)(\omega_C + \phi - 1)(y/k)}{\eta(\alpha + \beta)} |\Delta|^{-1} d\eta > 0, \quad (17)$$

and the change in steady-state per capita consumption is given by:

$$\tilde{c}(\infty) = \frac{(r - \alpha)(\alpha + \beta + \eta)(\phi\epsilon_K - \omega_I)(y/k)}{\eta(\alpha + \beta)} |\Delta|^{-1} d\eta > 0. \quad (18)$$

The impact effect on private consumption is unambiguously negative:

$$\tilde{c}(0) = \mathcal{L}\{\gamma_C, \lambda_2\} < 0, \quad (19)$$

where  $\gamma_C \equiv \frac{(r - \alpha)(\alpha + \beta + \eta)}{\eta(\alpha + \beta)} d\eta < 0$ .

### A.2.3 Constant Generational Turnover

If generational turnover is kept constant, the birth rate falls by less than the death rate, that is,  $d\eta = -\eta d\beta / (\alpha + \beta) < 0$ , where  $d\beta > 0$ , so that  $dn_N = (\alpha + \beta + \eta)(d\eta/\eta) < 0$ . The long-run effect on the per capita capital stock is negative:

$$\tilde{k}(\infty) = (\alpha + \beta + \eta) [r - \alpha - (r + \delta)(\phi - 1)] |\Delta|^{-1} \tilde{\eta} < 0, \quad (20)$$

if the labor supply effect dominates the generational turnover effect. The long-run change in per capita consumption is given by:

$$\tilde{c}(\infty) = (\alpha + \beta + \eta) [r - \alpha + (r + \delta)(1 - \phi\epsilon_K)] |\Delta|^{-1} \tilde{\eta} > 0, \quad (21)$$

and private consumption jumps up on impact:

$$\tilde{c}(0) = \frac{(\lambda_2 - \delta_{22})(\alpha + \beta + \eta)d\eta}{\delta_{12}\lambda_2} < 0. \quad (22)$$

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<sup>38</sup>See Heijdra and Ligthart (2004) for a formal derivation.

#### A.2.4 Fiscal Policy

A rise in public spending has qualitatively the same effects as a fall in the birth rate while keeping the rate of generational turnover constant. The steady state effect on the per capita capital stock is positive and is given by:

$$\tilde{k}(\infty) = \omega_G \left( \frac{y}{k} \right) [r - \alpha - (r + \delta)(\phi - 1)] |\Delta|^{-1} \tilde{g} > 0, \quad (23)$$

if the labor supply effect is sufficiently strong. The steady-state effect on private consumption is given by

$$\tilde{c}(\infty) = \omega_G \left( \frac{y}{k} \right) [r - \alpha + (r + \delta)(1 - \phi\epsilon_K)] |\Delta|^{-1} \tilde{g} < 0, \quad (24)$$

and per capita private consumption rises initially:

$$\tilde{c}(0) = -\frac{(\lambda_2 - \delta_{22})\omega_G \left( \frac{y}{k} \right) \tilde{g}}{\delta_{12}\lambda_2} < 0. \quad (25)$$

A rise in the capital income subsidy which can be represented by  $\tilde{\tau} < 0$  and  $\tau < 0$ ) yields an increase in the per capita capital stock:

$$\tilde{k}(\infty) = (r + \delta)(\omega_C + \phi - 1)(y/k) |\Delta|^{-1} \tilde{\tau} > 0, \quad (26)$$

and a rise in per capita consumption in the new steady state:

$$\tilde{c}(\infty) = (r + \delta)(\phi\epsilon_K - \omega_I)(y/k) |\Delta|^{-1} \tilde{\tau} > 0. \quad (27)$$

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